

Semester IV
CC - 9 (Modern Physics)

Sem-4 Unit-1: Short questions (AK)

1. (a) What are the characteristic features of photoelectric emission.
- (b) What is threshold wavelength for photo-emission. If the threshold wavelength increases when the emitting metal is changed, what can be said about the work function of the two metals?
- (c) Draw a graph between the frequency of light falling on a metal surface and the kinetic energy of the photoelectron emitted.
- (d) What is Compton wavelength of an electron? Is it a fundamental constant? Why? Find its value?
- (e) Show that it is not possible for a photon to transfer its entire energy to the recoil electron in Compton effect.
- (f) The work function for Zinc is 3.6 eV. The threshold frequency for the metal is 9×10^{14} Hz. Find the value of Planck constant.
- (g) A charged particle accelerated by 200 V has a de Broglie wavelength 0.20 \AA . Find the mass of the particle.
- (h) What are group velocity and phase velocity. Give the relation between the two.
- (i) State Heisenberg's uncertainty relation and hence show that electron cannot be constituent of atomic nucleus.
- (j) Show that the kinetic energy T of an electron, having de Broglie wavelength equal to Compton wavelength, is given by $T = m_0 c^2 (\sqrt{2} - 1)$ where m_0 is the rest mass of the electron.
- (k) Calculate the glancing angle at which electrons of 100 eV should fall on the lattice planes of a crystal to obtain a strong Bragg reflection in the first order. The lattice spacing is 2.15 \AA .
- (l) The life-time of an excited state of an atom is 10^{-8} s . Calculate the minimum uncertainty in the determination of the energy of the excited state.

① The energy density $P(\nu) d\nu$ of standing waves inside a cavity with metallic walls is given by $P(\nu) d\nu = \frac{8\pi\nu^2}{c^3} \bar{\epsilon} d\nu$ where $\bar{\epsilon}$ is the average energy of a standing wave.

(a) The classical theory assumes that a standing wave inside a cavity can have any value for the energy and therefore classical expression for average energy is given by

$$\bar{\epsilon} = \frac{\int_0^\infty \epsilon P(\epsilon) d\epsilon}{\int_0^\infty P(\epsilon) d\epsilon}$$

Using Boltzmann probability distribution show that $\bar{\epsilon} = kT$ and $P(\nu) d\nu = \frac{8\pi\nu^2 kT}{c^3} d\nu$.

(b) According to Planck hypothesis a standing wave inside a cavity can have only discrete energy values given by $\epsilon = n h \nu$, where h is the Planck's constant, ν is the frequency of the standing wave and $n = 0, 1, 2, \dots$ is an integer. Prove that, with Planck's hypothesis, the average energy of the standing wave is $\bar{\epsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$ and thus that the energy density is given by $P(\nu) d\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} d\nu$.

(c) Show that for small frequencies, the Planck's radiation formula reduces to the Rayleigh-Jeans formula.

② Stefan's law seeks to find a relation between the temperature T of the blackbody and the total radiance R_T , which is the total energy radiated out by the blackbody per unit area per unit time. R_T is defined as: $R_T = \int R(\nu) d\nu$

(a) Using the relation $R(\nu) = \frac{c}{4} P(\nu)$ and Planck law of radiation show that the total radiance depends on temperature T as $R_T = \sigma T^4$, where σ is called Stefan's - Boltzmann Constant.

(b) Find the numerical value of the Stefan's constant.

(c) Suppose an object is heated to about 3000 K. Of what color would this source appear - Reddish, Greenish, Bluish? (Use Wien's displacement law)

- 3 (a) What is photoelectric effect? Point out clearly the important features of the effect. How did Einstein explain them? Point out precisely the difference between Planck's hypothesis and Einstein's hypothesis.
- (b) The wavelength of the photoelectric threshold of tungsten is 2300 \AA . Determine the kinetic energy of the electrons ejected from the surface by ultraviolet light of wavelength 1800 \AA .
- 4 (a) The minimum electromagnetic energy that a human eye can detect is $1 \times 10^{-18} \text{ J}$. How many photons of 600 nm wavelength does that correspond to?
- (b) Draw a graph between the frequency of light falling on a metal surface and the kinetic energy of the photoelectrons emitted. How will this graph change if (i) the intensity of light is changed (ii) the metal is changed.
- (c) Show that it is impossible for a photon to give up all its energy and momentum to a free electron. This is the reason why the photoelectric effect can take place only when the photons strike bound electrons.
- 5 (a) The work function of sodium is 2.3 eV . We have two sources of light. First is intense, one watt HeNe laser at 633 nm and second is the touch light of mobile phone. Which one of the two sources has a finite probability of ejecting an electron from sodium and why?
- (b) Now, explain qualitatively how the energy and momentum conservation are simultaneously satisfied when photoelectric effect takes place with the bound electrons.
- (c) The maximum energy of photoelectrons emitted by a metal is E_1 when the incident radiation has a frequency ν_1 . If it be E_2 when the frequency is ν_2 , then show that Planck's constant $h = \frac{E_1 - E_2}{\nu_1 - \nu_2}$ ($E_1 > E_2$) and the work function ϕ of the metal $\phi = \frac{E_1 \nu_2 - E_2 \nu_1}{\nu_1 - \nu_2}$.

6 (a) What is Compton effect? What is its basic importance? Obtain an expression for the Compton shift. What is Compton wavelength?

(b) A photon of energy $h\nu$ is scattered through an angle $\pi/2$ by an electron initially at rest. The wavelength of the scattered photon is twice that of the incident photon. Find the frequency of the photon and the recoil angle of the electron.

7 (a) Show that if the recoil electron in Compton scattering has very low energy, the change in wavelength of the photon scattered at an angle ϕ is given by

$$\Delta\lambda = \frac{\lambda_0}{2} \left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} - 2\cos\phi \right), \text{ where } \lambda_0 \text{ is the Compton wavelength.}$$

(b) A photon of energy 511 keV is scattered through 90° . Applying the laws of conservation, show that the angle θ between the incident photon and the recoil electron satisfies the relation

$$\cos\theta - \sin\theta = \sqrt{5/4} \sin\theta \cos\theta.$$

8 (a) A photon of energy $h\nu$ is Compton-scattered through an angle α . Show that the ratio of the kinetic energy of the recoil electron to the energy of the photon is

$$\frac{\beta(1-\cos\alpha)}{1+\beta(1-\cos\alpha)} \text{ where } \beta = h\nu/m_0c^2.$$

(b) Show that in Compton scattering of a photon of energy $h\nu$ by an electron, initially at rest, the relation between the scattering angle θ of the photon and the recoil angle ϕ of the electron is given by

$$\cos\theta = 1 - \frac{2}{(1+\beta)^2 \tan^2\phi + 1} \text{ where } \beta = \frac{h\nu}{m_0c^2}$$

(c) A photon is scattered by a stationary free electron of mass m_0 . Find the momentum of the incident photon if the energy of the scattered photon is equal to the kinetic energy of the recoil electron with divergence angle 90° .

9 a) State de Broglie hypothesis. Using de-Broglie's hypothesis and non-relativistic considerations, calculate the wavelength in Angstrom of an electron in terms of the potential difference in volt through which the electron has been accelerated.

b) Show that the relativistic expression for the de-Broglie wavelength of an electron accelerated through a high p.d of V volt is

$$\lambda = \frac{h}{\sqrt{2m_0 eV}} \left(1 + \frac{eV}{2m_0 c^2} \right)^{-1/2}$$

with the usual significance of the symbol used.

c) Calculate the wavelength of an electron of energy
(i) 10 eV (ii) 100 kV.

10 a) Explain what you mean by 'matter waves'. Describe the neat diagrams, the experiments of Davisson and Germer on the diffraction of electrons. How does it establish the wave nature of matter.

b) Explain the difference between the group velocity and phase velocity. Derive an equation relating the two. Show that the phase velocity of de-Broglie wave is greater than the velocity of light in free space.

11 a) State and explain uncertainty principle. Explain it with two simple experiments. Show how you would obtain the uncertainty relationship from an analysis of the hypothetical experiments (i) diffraction of electron by a single slit (ii) detection of a γ -ray quantum by microscope.

b) Using uncertainty principle find out the zero point (minimum) energy of a 1-D linear harmonic oscillator.

c) Beta particles of kinetic energy 1 MeV are emitted from a radioactive nucleus of mass number 64. Prove, by the application of the uncertainty principle, that these particles can not exist inside the nucleus, given rest mass of electron is 0.5 MeV.

UNIT – 3 & 4

Short answer type Questions:

1. Verify that the rest mass energy of an electron is 0.51 MeV approximately.
2. Use the uncertainty relation to estimate the kinetic energy of a proton in the nucleus. Assume that the nuclear radius is about 5 fm. ($\hbar = 1.054589 \times 10^{-34}$ J.s)
3. Determine the radius of the ${}^{208}_{82}\text{Pb}$ nucleus. Assume $R_0 = 1.2$ fm.
4. What is mass defect and packing fraction?
5. Heavy nuclei emit α particles spontaneously. However, they are stable against the decay of a single proton or neutron or deuteron. Why is α emission favoured over these decays?
6. The half life of an unstable nucleus is τ . What fraction of the original nucleus will be decayed in time 3τ ?
7. A biological specimen (tree) after its death shows ${}^{14}\text{C}_6$ activity that is one-third of that of a living specimen. Estimate the age of the sample. The half life of ${}^{14}\text{C}_6$ is about 5500 years.
8. Explain the origin of fine structure in the spectrum of α particles emitted by radioactive nuclei.
9. Give one example each of β^- decay, β^+ decay and electron capture.
10. The cross section of a nuclear reaction is 10^2 barns – what is the significance of this statement?
11. What are prompt and delayed neutrons in a fission reaction?
12. Why is a moderator needed in a fission reactor?
13. What do you mean by temporal and spatial coherence?
14. How do you interpret high monochromaticity and high directionality of laser radiation?
15. Explain the concept of negative temperature in connection with the population inversion in an active medium.

Long answer type questions:

1. Estimate the density of nuclear matter in terms of (a) kg/m^3 and (b) nucleons/ fm^3 .
2. Calculate the binding energy of the ${}^{39}_{19}\text{K}$ nucleus.
3. What are mirror nuclei? Calculate the difference in Coulomb energies of the mirror nuclei.
4. (a) Calculate the difference in binding energies of ${}^{15}_8\text{O}$ and ${}^{15}_7\text{N}$. Given: $M({}^{15}_8\text{O}) = 15.003065$ u, $M({}^{15}_7\text{N}) = 15.000109$ u, $M({}^1_1\text{H}) = 1.007825$ u, $M_n = 1.008665$ u
(b) Assuming the difference in binding energy is equal to the difference in Coulomb energy, calculate the nuclear radius of ${}^{15}_8\text{O}$ and ${}^{15}_7\text{N}$.
5. Why does the binding energy per nucleon for medium-sized nuclei remain relatively constant? How do you explain the fall of the binding energy curve for lighter as well as heavier nuclei? Explain from the binding energy curve why energy is released in fission and fusion.
6. Lighter nuclei generally have $N \approx Z$, but for massive stable nuclei, N is always greater than Z. Explain why.
7. Find out the approximate relation between the kinetic energy of emitted α particle and the disintegration energy in terms of the atomic no of parent nucleus.
The nuclear masses of a parent and its daughter are 226.025 and 222.017 amu respectively. The mass of the α particle is 4.002 amu. Find out the Q value and the kinetic energy of the α particle emitted.
8. The energies of the α particle emitted in the decays of ${}^{226}\text{Ra}_{88}$ and ${}^{226}\text{Th}_{90}$ are 4.9 MeV and 6.5 MeV, respectively. Find the ratio of their half lives.
9. (a) Calculate the height of the Coulomb barrier faced by an α particle while trying to escape from ${}^{238}\text{U}$. The highest energy of α particles emitted in the decay of ${}^{238}\text{U}_{92}$ to the ${}^{234}\text{Th}_{90}$ nucleus is 4196 keV. Is there an apparent anomaly between this and your calculated value? If so, how is the anomaly resolved? (b) Assuming a square potential barrier, obtain an expression connecting the decay probability (or the transmission coefficient) and the energy of an α particle.

10. Show that if positron decay is energetically allowed for the nucleus of a neutral atom, electron capture is also allowed, but not vice-versa.
11. Explain why the postulate about the existence of the neutrino was necessary to remove the difficulties in the interpretation of β decay. Explain qualitatively how the hypothesis of neutrino solves the apparent breakdown of the conservation principles of momentum and energy in β decay.
12. What do you mean by end point energy of the emitted particles in β decay? Tritium emits negative β particles. Calculate the end point energy of the β particles from the following data. Mass of Tritium = 3.01695 amu, mass of ${}^3\text{He}$ = 3.01693 amu.
13. ${}_{88}^{226}\text{Ra}$ decays to ${}_{86}^{222}\text{Rn}$ by emitting three groups of α particles of energies 4.777, 4.503 and 4.342 MeV. Determine the energies of the associated γ rays.
14. Give a simple explanation of nuclear fission on the basis of liquid-drop model of the nucleus.
15. ${}^{235}\text{U}$ is bombarded by thermal neutrons and fission occurs. The fragmented products are ${}^{141}\text{Ba}_{56}$ and ${}^{92}\text{Kr}_{36}$. Three prompt neutrons are released when fission occurs. Write the nuclear reaction involved . Calculate the Q value of the reaction and hence calculate the energy released from the fission of 1 g of ${}^{235}\text{U}$.
16. Obtain the four factor formula for a fission reactor.
17. Discuss briefly the principles of thermo-nuclear fusion.
18. Describe with energy level diagrams the phenomena of spontaneous emission, stimulated emission and stimulated absorption in a two level system.
19. What is population inversion? How is it obtained in practical lasers?
20. What are Einstein's A, B coefficients? Derive a relation between them.
21. What is an optical resonator? Discuss the role played by it in a laser system.
22. Derive threshold condition for laser action.
23. Draw a neat sketch of Ruby laser. With the help of a simple energy level diagram describe the operation of a ruby laser.
24. Give the construction of a He-Ne laser. With the help of a simple energy level diagram, show how population inversion is activated here.